FIELD OF THE INVENTION

The present invention relates to a method for forming more rapidly a stochastic numerical model of Gaussian or related type, representative of a porous heterogeneous medium (such as a hydrocarbon reservoir for example) calibrated in relation to data

5 referred to as dynamic, characteristic of the displacement of fluids in the medium such as, for example, production data (pressures obtained from well tests, flow rates, etc.).

The method according to the invention finds applications in the sphere of underground zone modelling intended to generate representations showing how a certain physical quantity is distributed in a zone of the subsoil (permeability notably),

10 best compatible with observed or measured data, in order for example to favour the development thereof.

BACKGROUND OF THE INVENTION

Optimization in a stochastic context consists in determining realizations of a stochastic model which meet a set of data observed in the field. In reservoir

15 engineering, the realizations to be identified correspond to representations, in the reservoir field, of the distribution of carrying properties such as the permeability or porosity. These realizations form numerical reservoir models. The available data are, for example, isolated permeability or porosity measurements, a spatial variability model determined according to isolated measurements or data directly related to the fluid

20 flows in an underground reservoir, i.e. pressures, breakthrough times, flow rates, etc. The latter are often not linearly related to the physical properties to be modelled. A randomly drawn realization is generally not in accordance with the whole of the data

collected. Coherence in relation to the data is integrated in the model by means of an inverse procedure :

- Tarantola, A., "Inverse problem theory – Methods for data fitting and model parameter estimation", Elsevier Science Publishers, 1987.

5   The simplest technique is therefore the trial-and-error method. This approach consists in randomly taking realizations until a realization meeting the data is obtained. It affords the advantage of conservation of the spatial variability model but it requires a prohibitive calculating time. It is therefore very rarely used in practice.

An option that is often preferred is based on the gradients calculation. The gradients

10   methods allow to modify an initial realization in a direction of search which is estimated from the gradients. The modifications are applied iteratively until data calibration is considered to be acceptable. The gradients methods are attractive because of their efficiency. However, they are no longer suitable when the number of parameters, i.e. the number of permeability and porosity values forming the numerical model, is large.

15   Besides, they do not allow to modify the realizations while respecting the spatial structure of the stochastic model.

More recently, a geostatistical parameterization technique has been introduced to constrain, by gradual deformation, the stochastic realizations to data on which they depend non-linearly. It is described in patents FR-2,780,798 and FR-2,795,841 filed by

20   the applicant, and in the following publications, notably :

- Hu, L.Y., 2000, Gradual deformation and iterative calibration of Gaussian-related stochastic models : Math. Geology, Vol.32, No.1,

- Le Ravalec, M. Et al., 2000, The FFT moving average (FFT-MA) generator : An efficient numercial method for generating and conditioning Gaussian simulations : Math. Geology, Vol.32, No.6,

- Hu, L.Y., Blanc, G. And Noetinger, B. (2001) : Gradual deformation and iterative calibration of sequential stochastic simulations. Math. Geology, Vol.33, No.4.

This method has been successfully applied in various cases, notably from data obtained from oil development fields, as described in the following documents :

- Roggero, F. et al., 1998, Gradual deformation of continuous geostatistical models for history matching, paper SPE 49004 : Proc. SPE Annual Technical Conference and Exhibition, New Orleans,

- Hu, L.Y. et al., 1998, Constraining a reservoir facies model to dynamic data using a gradual deformation method, paper B-01 : Proc. 6th European Conference on Mathematics of Oil Recovery (ECMOR VI), 8-11 September 1998, Peebles, Scotland.

As reminded in detail hereafter, the gradual deformation method allows to gradually modify a realization of a stochastic model of Gaussian or Gaussian-related type while respecting the spatial structure of the model.

Stochastic optimization

Let $f^{obs} = (f_1^{obs}, f_2^{obs},..., f_M^{obs})$ be the field data and $f = (f_1, f_2,..., f_M)$ the corresponding responses simulated for a realization z of a given stochastic model Z. In general, the responses $f = (f_1, f_2,..., f_M)$ are obtained by solving numerically the direct problem. Thus, if z represents a permeability field, data f can be pressure measurements. In this case, they are simulated from a flow simulator. The goal of a

stochastic optimization is to produce realizations of Z which reduce the differences between the observed data and the numerically simulated corresponding responses. These differences are measured by the following objective function :

$$J = \frac{1}{2}\sum_{m=1}^{M} \omega_m (f_m - f_m^{obs})^2$$

5   Coefficients ω_m are weights assigned to data f_m^{obs}. f_m are functions of realization z. In this sense, minimization of the objective function is a problem with several variables.

Let N be the number of grid cells or of components forming realization z. N is often very large ($10^4 \sim 10^7$). It is therefore very difficult to perform an optimization directly in relation to the components of z. Furthermore, realization z, even modified, must remain

10   a realization of Z. The gradual deformation method allows these difficulties to be overcome.

Random search from the gradual deformation method

We now consider a random field Z that can be transformed into a Gaussian random field Y. The gradual deformation technique allows to construct a continuous chain of

15   realizations by combining an initial realization y_0 of Y with another realization u_1, referred to as complementary, of Y, u_1 being independent of y_0 (Figure 1a). The combination coefficients are for example cos(t) and sin(t), and the combined realization meets the relation :

y(t) = y_0 cos t + u_1 sin t

20   where t is the deformation parameter.

Another realization chain construction technique consists in combining the initial realization no longer with one, but with P complementary realizations u_p (p=1,P) of Y

(Figure 1b). The coefficients of the combination are such that the sum of their squares is

1.

Once the chain is formed, it can be explored by varying deformation parameter t and one tries to identify, from among all the realizations of this chain, the realization which minimizes the objective function. This minimization is performed in relation to t. Parameterization according to the gradual deformation method allows to reduce the number of dimensions of the problem from N to 1, where N is the number of values that constitute the field to be constrained. Furthermore, the sum of the combination coefficients squared being 1, the optimized realization $y(t_{opt})$ still is a realization of Y : it follows the same spatial variability model as all the realizations of Y.

However, if the exploration of the realizations space is restricted to a single chain, our possibilities of sufficiently reducing the objective function are greatly limited. The above procedure therefore has to be repeated, but with new realization chains. These realization chains are constructed successively by combining an initial realization which is here the optimum realization determined at the previous iteration, with a complementary realization of Y, randomly drawn each time. Thus, at iteration l, the continuous realization chain is written as follows :

$$y_l(t) = y_{l-1} \cos t + u_l \sin t .$$

y_{l-1} is the optimum realization defined at iteration l-1 and the u_l are independent realizations of Y. The latter are also independent of y_0. This formulation implies that the realization chain corresponds to a hyperellipse of dimension N.

Minimizing the objective function in relation to t allows to improve or at least to preserve calibration of the data each time a new realization chain is explored. This

iterative minimum search procedure is continued as long as data calibration is not satisfactory. The random side of the method lies in the fact that, upon each iteration, the complementary realization is drawn at random. In fact, the direction of search that is followed from the optimized realization at the previous stage is random. The direction

5   of search, for a given chain and from the optimum realization defined above, is :

$$\frac{dy_I(t)}{dt}\bigg)_0 = -y_{I-1}\sin 0 + u_I \cos 0$$
$$= u_I$$

This direction of search only depends on u_I. Furthermore, u_I being independent of the complementary realizations already generated u_1, u_2,..., u_{I-1} and also of y_0, the direction of search at the start of each new chain is orthogonal to the tangent defined for

10   the previous chain at the same point (Figure 2). Although it may seem appropriate to select a direction of search orthogonal to this tangent, there is an infinity of possible orthogonal directions.

Experience shows that, after several iterations, the new directions of search no longer contribute significantly to the decrease of the objective function (Figure 6).

15   It has also been considered combining the initial realization not only with one, but with several complementary realizations. In this case, the number of deformation parameters increases : it is equal to the number of complementary realizations involved in a gradual combination. Although the optimization process is then more flexible, several parameters have to be managed, which is not easy. Besides, such a process is

20   not necessarily more efficient because it can depend on the execution of a larger number of direct flow simulations.

SUMMARY OF THE INVENTION

The method according to the invention allows more rapid formation of a numerical

model representative of the distribution of a physical quantity in a porous heterogeneous

medium such as an underground zone (oil reservoirs, aquifers, etc.), constrained by data

5   collected in the medium (dynamic data characteristic of the displacement of fluids in the

medium), collected by measurements (in production, injection or observation wells for

example) or previous observations.

It comprises an iterative process of gradual deformation wherein an initial

realization of at least part of the selected model of the medium is linearly combined

10   with at least a second realization independent of the initial realization, the coefficients

of this linear combination being such that the sum of their squares is 1, and an objective

function measuring the difference between a set of non-linear data deduced from said

combination by means of a medium simulator and said geologic and dynamic data is

minimized by adjusting the coefficients of the combination, the iterative process being

15   repeated until an optimum realization of the stochastic model is obtained.

The method is characterized in that the rate of gradual deformation to the optimum

model representative of the medium is accelerated by selecting as the second realization

to be combined with the initial realization at least one composite realization obtained by

selecting beforehand a direction of descent defined as a function of the gradients of the

20   objective function in relation to all the components of said initial realization.

The composite realization is obtained for example by linear combination of a set of

independent realizations of the model, the coefficients of the combination being

calculated so that the direction of descent from the initial realization y is as close as

possible to the one defined by the gradients of the objective function in relation to all the components of the initial realization.

Optimization is for example carried out from a deformation parameter which controls the combination between the initial realization and the composite realization.

5   In cases where said combination affects only part of the initial realization, the iterative process of gradual deformation is applied to a Gaussian white noise used to generate a Gaussian realization and the derivatives of the objective function with respect to the components of the Gaussian white noise are determined.

According to an implementation mode, the initial realization is combined with a 10   certain number M of composite realizations, all obtained by composition from P_m independent realizations of Y, the optimization involving M parameters.

In other words, the method essentially comprises a new gradual combination scheme which takes account of the information provided by the gradients at the initial point of any chain of realizations. Construction of a chain always starts from an initial 15   realization and from a set of complementary realizations, all independent and coming from the same stochastic model. The initial realization is however not directly combined with the complementary realizations. The complementary realizations make it possible to explore the realizations space in different directions. These directions are not equivalent : some allow to get nearer to the optimum. At this stage, a realization 20   referred to as composite realization is elaborated by combining the complementary realizations only. A chain of realizations is then created from the initial realization and from this composite realization. This chain, like the chain proposed in the basic case of gradual deformation, can be explored by means of a single deformation parameter.

The composite realization is constructed so as to propose a direction of search along the chain as close as possible to the direction of descent given by the gradients. As mentioned above, all complementary realizations are not equivalent : the composite realization takes the best of each complementary realization.

5   The method allows to reach the formation of the numerical model representative of the medium more rapidly.

BRIEF DESCRIPTION OF THE FIGURES

Other features and advantages of the method according to the invention will be clear from reading the description hereafter of an application given by way of non

10   limitative example, with reference to the accompanying drawings wherein :

- Figures 1a, 1b show gradual deformation schemes (referred to as GDM) that are already known,

- Figure 1c shows the gradual deformation scheme (GBC) corresponding to the method according to the invention,

15   - Figure 2 shows realization chains in a Euclidean space with N dimensions, where the tangent at the level of the optimized realization for a realization chain l-1 (RC_{l-1}) is orthogonal to the direction of search for the initial realization of the next realization chain l (RC_l),

- Figure 3 shows the projection of the direction of descent v in the subspace U defined

20   by the orthonormal base formed by P independent realizations (u_1,..., u_p),

- Figures 4A to 4C show permeability distribution examples respectively for the reference realization, the initial realization and the realization constrained to the pressure data,

- Figures 5A to 5E respectively show the variations as a function of time of the bottomhole pressures respectively simulated for the five wells (BHP-OBS1, BHP-OBS2, BHP-PRO1, BHP-OBS3, BHP-OBS4) of Figures 4, respectively for the reference (data), initial and constrained (match) permeability distributions, and

- Figure 6 shows the evolution (OF) of the objective function as a function of the number k of flow simulations performed, GDM1 corresponding to an optimization carried out by combining an initial realization and a single complementary realization, GDMGBC3, to an optimization carried out by combining the initial realization and a composite realization constructed from three complementary realizations, GDMGBC10, to the optimization carried out by combining the initial realization and a composite realization constructed from ten complementary realizations, and GDMGBC30, to an optimization carried out by combining the initial realization and a composite realization constructed from thirty complementary realizations.

DETAILED DESCRIPTION

The method according to the invention allows to orient upon each iteration the construction of the realization chain in order to reach a desirable direction of search. The technique selected takes advantage of the information provided by the direction of descent defined by the gradients of the objective function J. It can be implemented by means of a numerical simulator of a type known to the man skilled in the art, such as the ATHOS or ECLIPSE simulators.

Search oriented by the direction of descent

We consider at this stage the direction of descent (evolution towards a minimum value, whether local or not) defined by the gradients of the objective function J in relation to all the components of realization z. These gradients are deduced from

5   sensitivity coefficients $\partial f_m / \partial z_n$:

$$\frac{\partial J}{\partial z_n} = \sum_{m=1}^{M} \omega_m (f_m - f_m^{obs}) \frac{\partial f_m}{\partial z_n}$$

The problem of sensitivity coefficients calculation has been widely dealt with in the scientific literature. The following document can for example be referred to :

- Sun, N.Z., Inverse problems in groundwater modelling, Kluwer Acad. Publ.,

10   Dordrecht, The Netherlands, 1994.

In particular, the adjoint state technique allows to calculate all of these coefficients by solving the direct problem and its adjoint problem, within a time interval equivalent to twice the time required for solution of the direct problem.

Since the goal is to integrate the information provided by the gradients in the

15   gradual deformation method, we have to go back to the Gaussian realization which results from the transformation of z. Two cases can be distinguished according to whether isolated conditioning data, i.e. measurements of z at certain points, are available or not.

When no data are available, if y is the Gaussian realization obtained by

20   transforming z, then :

$$\frac{\partial J}{\partial y_n} = \frac{\partial J}{\partial z_n} \frac{dz_n}{dy_n}.$$

In the opposite case, we assume that z is known at certain points α. Thus, the Gaussian realization s, deduced from the transformation of z, is a conditional realization of Y. To obtain s, we generate a non-conditional realization y of Y and condition it to the values known at α by kriging. The conditional realization is deduced from :

5        $s = s^* + (y - y^*)$.

s* and y* are respectively calculated by kriging from the real data and the data of y generated at the level of points α. We can then show that :

$$\frac{\partial J}{\partial y_n} = \begin{cases} \dfrac{\partial J}{\partial z_n}\dfrac{dz_n}{ds_n}, \forall n \neq \alpha \\ 0, \qquad \forall n = \alpha \end{cases}$$

For continuous physical properties, dz_n/dy_n or dz_n/ds_n are calculated from the

10   anamorphosis function allowing to transform realization z into a Gaussian realization. When the physical properties considered are category or discrete properties, these derivatives do not exist. The gradients techniques then cannot be applied.

These various relations are of direct interest if a realization just has to be deformed globally. On the other hand, if it is desired to deform it locally, the gradual deformation

15   method has to be applied to the Gaussian white noise x used to generate y. In this case, an additonal stage is required : calculation of the derivatives of the objective function with respect to the components of the Gaussian white noise.

To illustrate the calculation of these derivatives, we suggest to concentrate on the particular case where the non-conditional Gaussian realization y is obtained from the

20   FFT-MA generator described in the article published by Le Ravalec et al. 2000 mentioned above.

The basic principle of this FFT-MA (FFT-Moving Average) generator is to transform a Gaussian white noise x into a Gaussian realization y correlated from a convolution product :

y = g * x.

5   Function g results from the expansion of covariance function C such that C = g * g^t, where g^t is the transpose of g. The derivatives of the objective function with respect to the components of the Gaussian white noise are :

$$\frac{\partial J}{\partial x_n} = \sum_l \frac{\partial J}{\partial y_l} \frac{\partial y_l}{\partial x_n}$$

The discrete expression for the convolution product leads to $y_l = \sum_k g_{l-k} x_k$, which

10   implies $\partial y_l / \partial x_n = g_{l-n}$. If we introduce this formulation in the derivatives of the objective function, we show that :

$$\frac{\partial J}{\partial x_n} = \sum_l \frac{\partial J}{\partial y_l} g_{l-n} = \left(\frac{\partial J}{\partial y} * g \right)_n$$

This formula expresses the fact that the derivative of the objective function with respect to the nth component of the Gaussian white noise is given by the nth component of the field obtained by convoluting all the derivatives of the objective function with respect to the components of the Gaussian realization with the kernel of the covariance function. From the framework established for the FFT-MA generator, these derivatives are determined as follows.

     1-Calculation of the Fourier transform of $\partial J / \partial y$, these derivatives being obtained

20   by means of the direct numerical simulator ;

2-Multiplication of this Fourier transform with that of g that is provided by FFT-MA ;

3-Calculation of the inverse Fourier transform of the previous product.

The time required for calculation of these derivatives is negligible : it represents an additional time of 2/3 in relation to the simulation of a Gaussian realization by FFT-MA.

Whatever the realization generator and the direct numerical simulator, we assume hereafter that we can define a direction of descent from the derivatives of the objective function. If optimization of the objective function is performed in relation to this direction of descent only, the coherence of the realization in relation to the spatial variability model is generally destroyed. In the section hereunder, we integrate the information provided by the derivatives of the objective function in the gradual combination scheme.

Taking account of the derivatives of the objective function in the gradual deformation process

We consider the realization chain $y_1(t)$ constructed from y_0 and from another realization u of Y (Figure 1a). Now, instead of using a complementary realization u as it is, we randomly draw P complementary realizations u_p (p=1,2,...,P) of Y and we substitute u for a combination of the $u_{p=1,P}$ (Figure 1c). This combination is not any combination : it follows the following construction mode :

$$u = \sum_p \lambda_p u_p \ \textit{with} \ \sum_p \lambda_p^2 = 1 \qquad (1)$$

The resulting realization u is a realization of Y and it is independent of y_0. According to Eq.1, u is also the direction of search calculated for chain $y_1(t)$ at the starting point y_0. We therefore suggest to construct u so that direction u is as close as possible to the direction of descent given by the gradients at y_0.

5   We first define the space V of the vectors with N dimensions provided with the scalar product :

$$\langle y_i, y_j \rangle = \frac{1}{N} \sum_{n=1}^{N} y_{i,n} y_{j,n} \quad \forall y_i, y_j \in V$$

$y_{i,n}$ and $y_{j,n}$ are respectively the nth components of vectors y_i and y_j. Let U be a subspace of V defined by the orthonormal base $(u_1, u_2, ..., u_p)$. The direction of search in U

10   which is the closest to the direction of descent v is given by the projection of v in U (Figure 3) :

$$v_U = \sum_p \langle v, u_p \rangle u_p$$

By normalizing this vector, we obtain the desired direction u. The combination coefficients λ of Eq.(2) are thus :

15     $$\lambda_p = \langle v, u_p \rangle \Big/ \sqrt{\sum_{q=1}^{P} \langle v, u_q \rangle^2} \quad p = 1, P$$

The realization u thus defined is referred to as composite realization.

We have considered so far the construction of realization chains by combination of the initial realization with a composite realization. This technique can however be generalized to the construction of chains involving a certain number M of composite

20   realizations, all obtained by composition from P_m realizations of Y, these $\sum_{m=1}^{M} P_m$ realizations being independent. Optimization then involves M parameters. This

technique increases the degree of freedom in the optimization process, but it requires management of M optimization parameters.

Numerical example

We construct a synthetic reservoir model on which the method according to the invention is tested.

The synthetic reference reservoir is shown in Figure 4A. It is a monolayer reservoir comprising 51x51 grid cells which are 10 m thick and 40 m in side. The permeability distribution is lognormal with an average of 200 mD and a standard deviation of 100 mD. The logarithm of the permeability field is characterized by an isotropic spherical variogram and a correlation length of 480 m. The other petrophysical properties are constant : the porosity is 25 %, the total compressibility 5.10^{-4} bar^{-1} and the fluid viscosity 1 cP. A production well BHP-PRO1, with a radius of 7.85 cm, free of any skin effect, is at the centre of the reservoir : it is surrounded by four observation wells (BHP-OBS1, BHP-OBS2, BHP-OBS3, BHP-OBS4) (Figures 4). A numerical flow simulation allows to obtain, for this reservoir, a set of reference data comprising the bottomhole pressures of the five wells (Figure 5).

The object of the inverse problem is to determine a reservoir model coherent with the pressure data, the permeabilities distribution being assumed to be unknown. Four optimization processes are therefore launched, starting from the same initial realization (Figure 4B). For each process, we consider a single optimization parameter, i.e. the deformation parameter. The first process (GDM1) takes up the conventional gradual deformation scheme with construction of a realization chain using the initial realization and a single complementary realization. The other three processes (GDMGBC3,

GDMGBC10 and GDMGBC30) illustrate the application of the method according to the invention : the chains are in this case elaborated with the initial realization and a composite realization obtained from the combination of 3, 10 and 30 complementary realizations. The composite realization is constructed as explained above (Eq.2) by integrating the information provided by the gradients of the objective function with respect to the Gaussian white noise. For each process, the evolution of the objective function (OF) as a function of the number k of flow simulations performed is shown in Figure 6.

It can be observed that using the gradients and increasing the number of complementary realizations allows the objective function to decrease more rapidly for the same number of flow simulations.